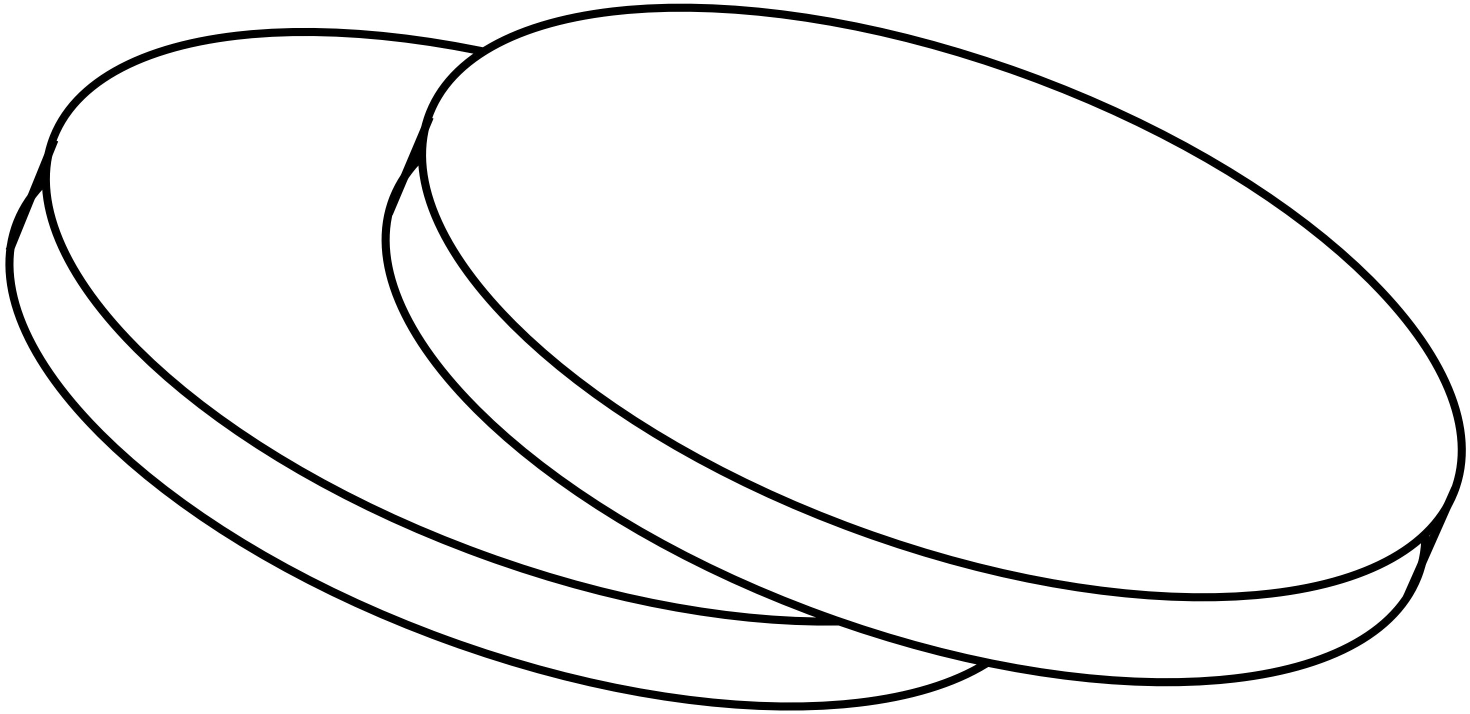
Plasma Instabilities in the QGP

Guy Moore: Arnold, Lenaghan, Yaffe
Mrówczyński, Strickland, Rebhan, Romatschke, Venugopalan, Dumitru, Nara, etc.

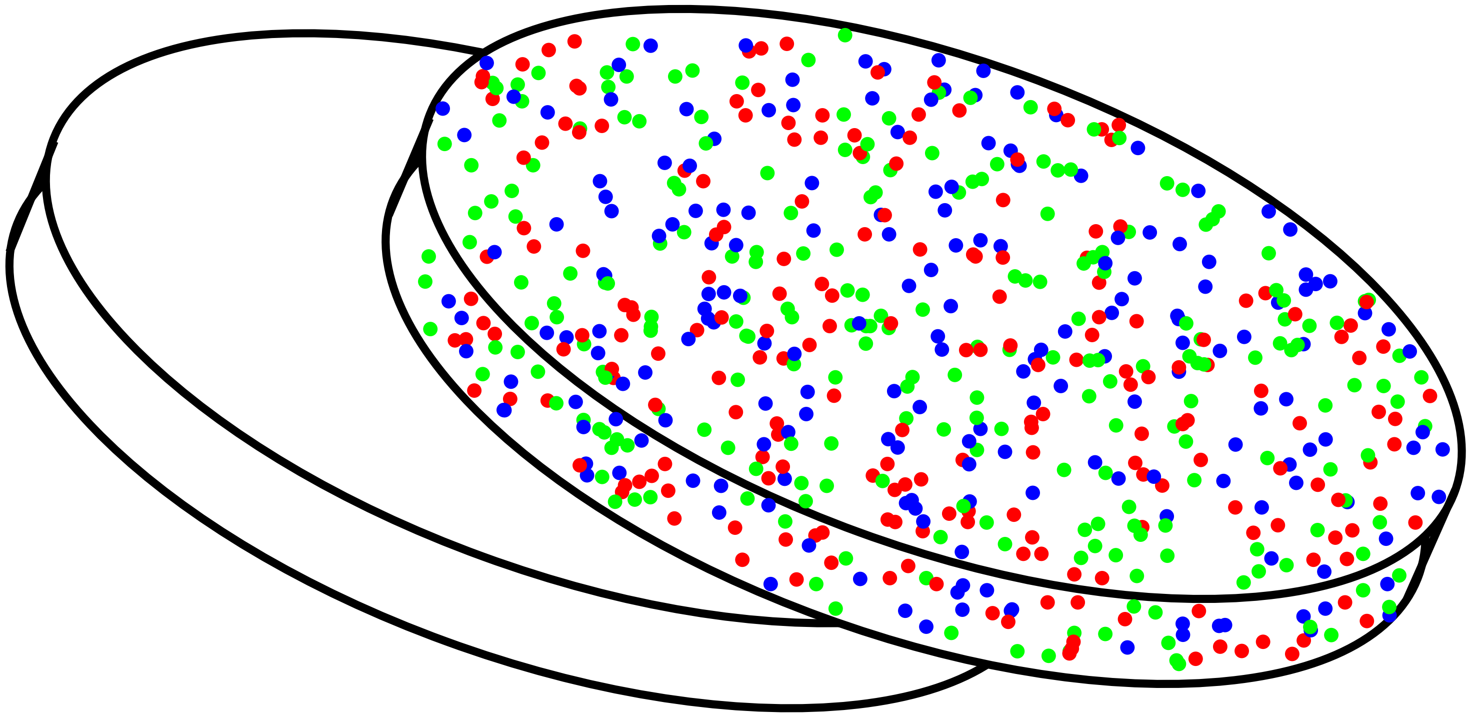
1. Physics of Weibel instability
2. Hard loop approach
3. Lattice implementation
4. Why to Expect Saturation
5. Numerical Results
6. What I expect for strong anisotropy

Just before collision



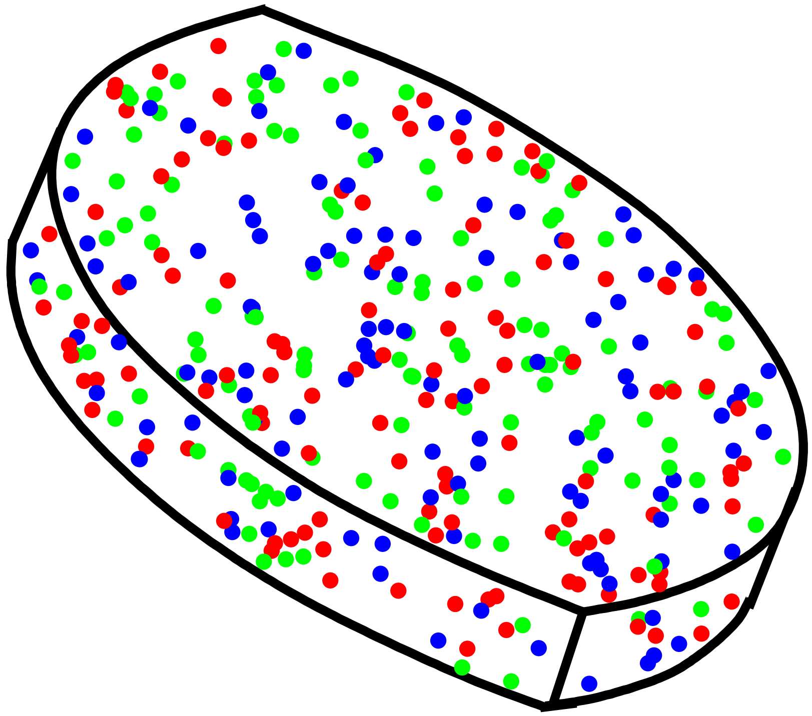
Lorentz contracted nuclei, striking with nonzero impact parameter

Each nucleus is ~ 200 p, n, each built of ~ 50 $q, \bar{q}, g$



It is the $q, \bar{q}, g$ which scatter.

Region where Collision Occurs:



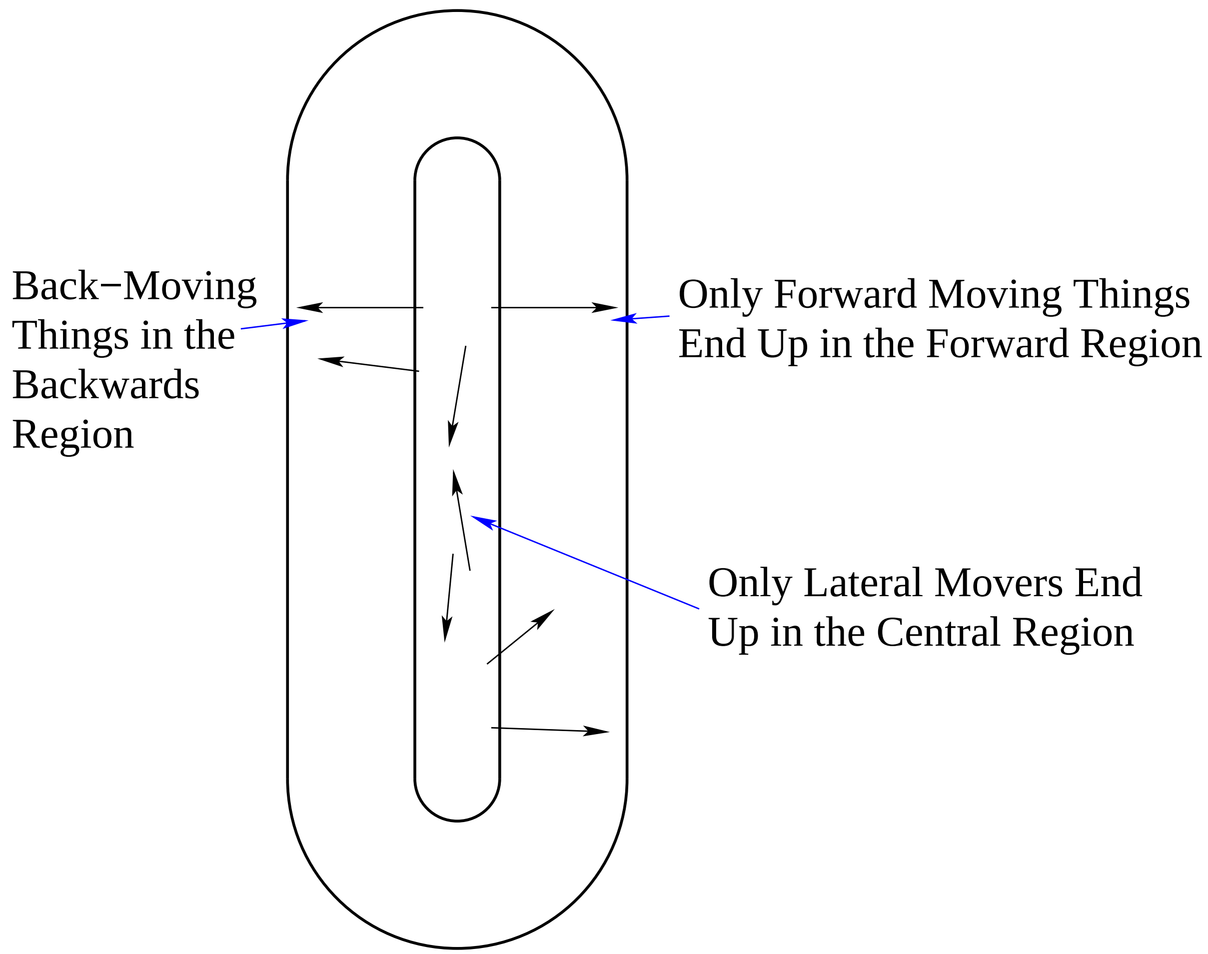
Irregular “flat almond” shaped overlap region:

Longer than it is wide, Wider than it is thick

~ 2000 partons, p distribution azimuthally isotropic

Momentum Selection

Side-on view of the flat almond as it expands



Development of anisotropy

The QGP becomes anisotropic unless scattering is very efficient.

Large, high energy nuclei $\Rightarrow$ high energy density

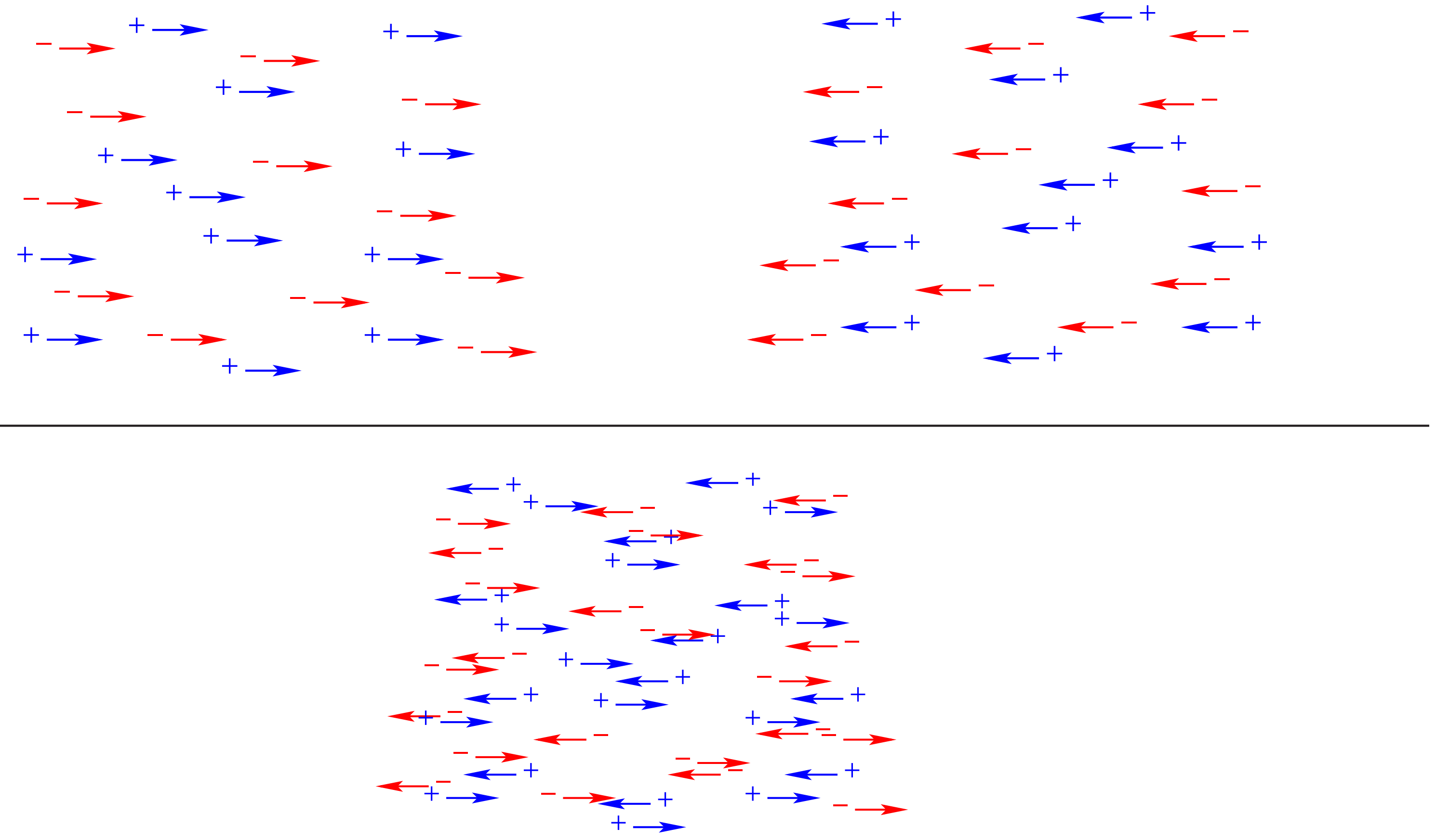
High energy density $\Rightarrow$ $\alpha_{\mathrm{s}}(\epsilon^{1/4})$ small

Weak coupling $\Rightarrow$ scattering is *not* efficient.

Expect anisotropy in limit of high energy collision

Simplest to understand anisotropic system

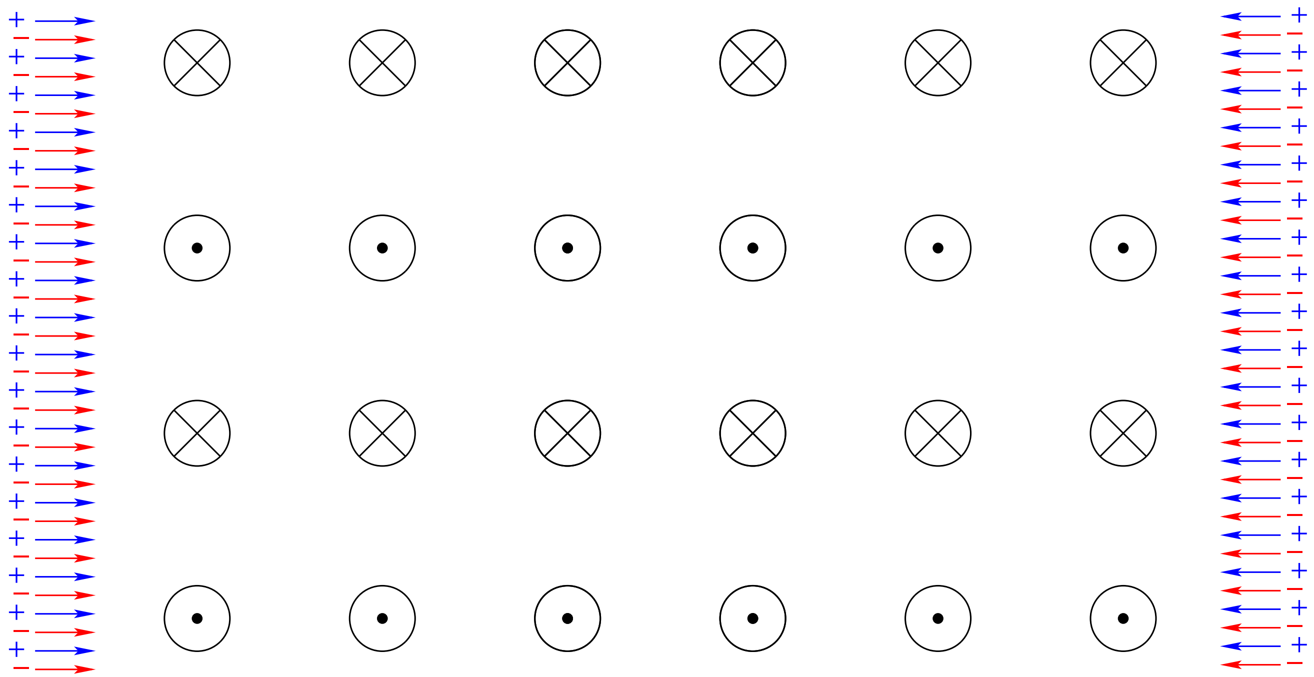
Interpenetrating plasmas



What happens?

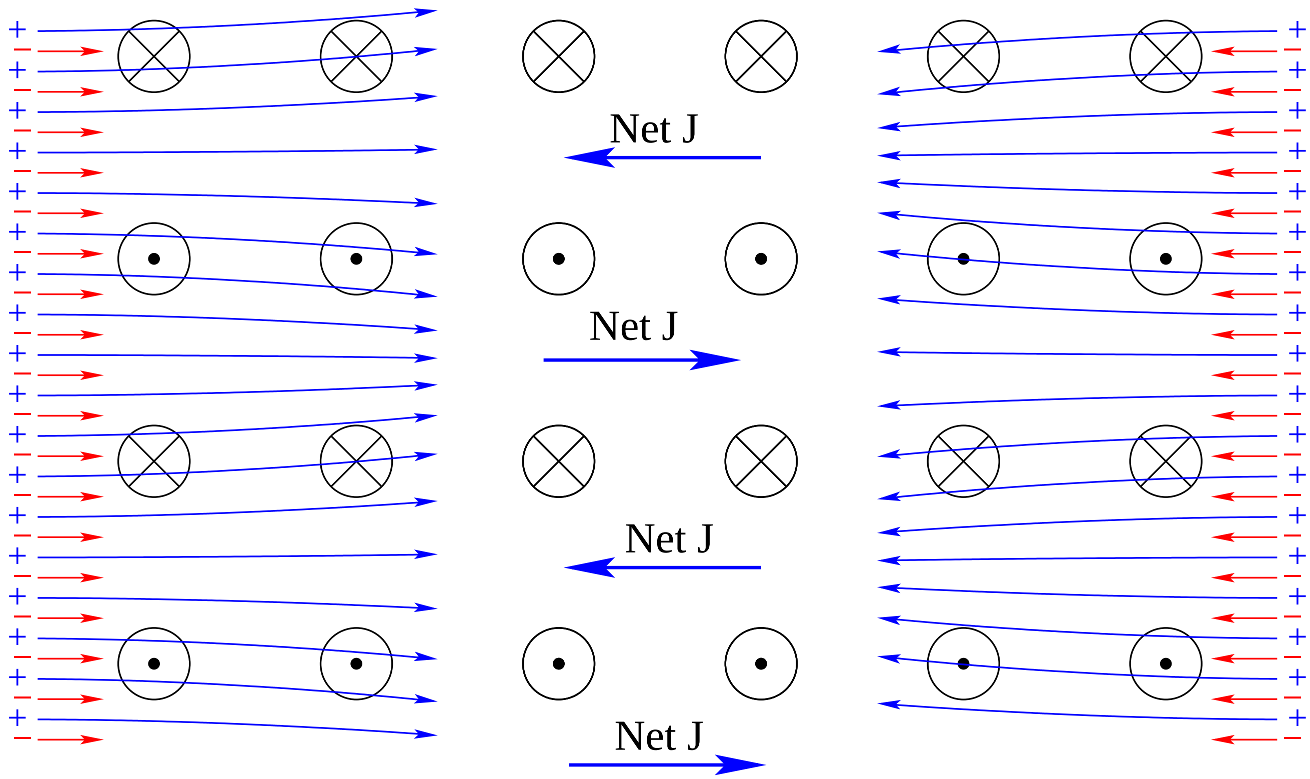
Magnetic field growth!

Consider the effects of a seed magnetic field $\hat{B} \cdot \hat{p} = 0$ and $\hat{k} \cdot \hat{p} = 0$



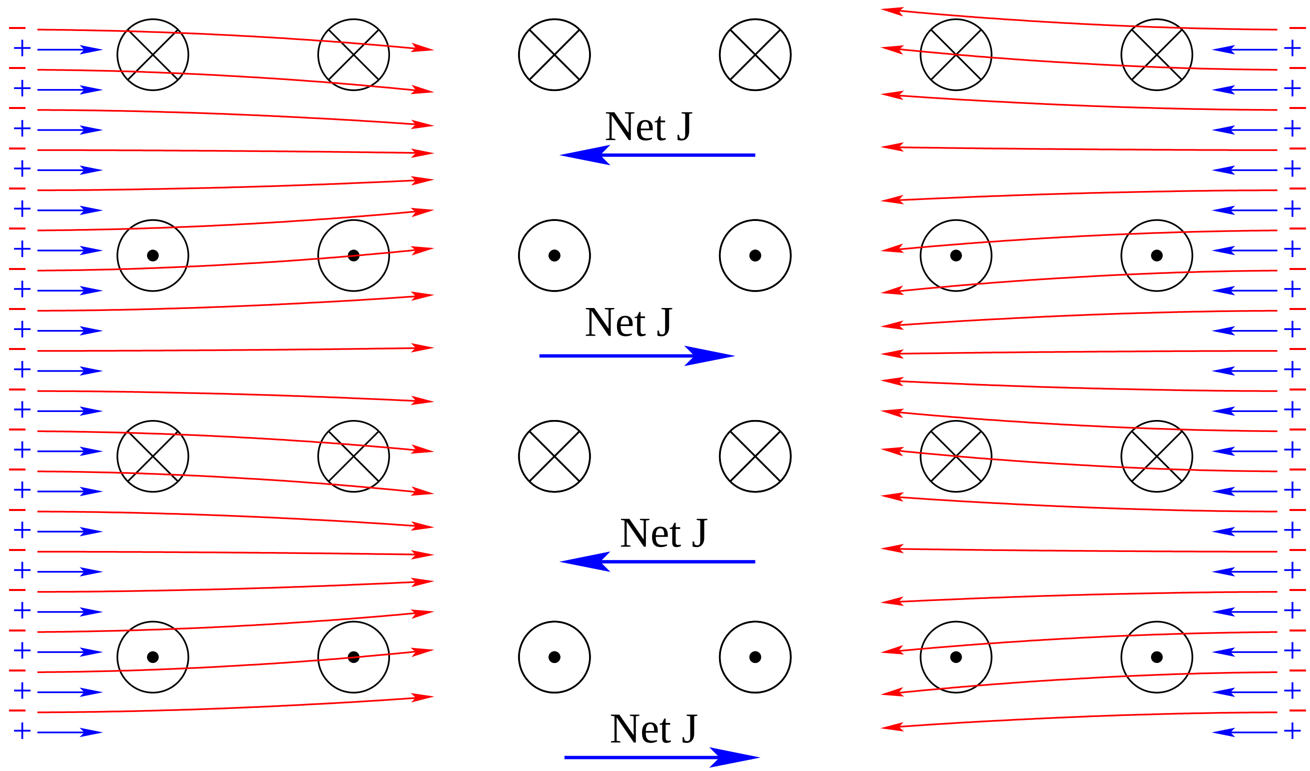
How do the particles deflect?

Positive charges:



No net ρ. Net current is induced as indicated.

Negative charges:



Induced B *adds* to seed B. Exponential Weibel instability

Linearized analysis: B grows until bending angles become large.

How? What about thermodynamics?

No instability occurs for isotropic plasmas. The effect of one direction of charge, just seen, is canceled by effects of other directions of charges.

Anisotropic plasma–low entropy, large available free energy. Maximize entropy by deflecting charges to be more isotropic. E, B fields do exactly that–and are most efficient method leading to isotropy and equilibration.

Weibel instability: $Always$ present for any anisotropy.

How fast is Weibel instability?

Weibel instability is a kind of "plasma screening."

Gauge fields of wavenumber k bend particles, $F = gv \times B$

$$\delta p \sim g\frac{B}{k} \sim gA$$

inducing a current $j \sim n\delta p/p$ which is important if

$$\nabla \times B \sim gj \quad \Rightarrow \quad k^2 A \sim gj \sim \frac{g^2 n}{p} A \quad \Rightarrow \quad k \sim g\sqrt{\frac{n}{p}} \quad \text{(think } gT\text{)}$$

Exponentiation time is $\sim g\sqrt{n/p}$ ("gT")

Hard Loop Expansion

General method to treat plasma screening effects. Valid if

1. Weak coupling $\alpha_{\rm s} \ll 1$
2. Separation of scale $p \gg k$ (or $n \ll p^4/g^2$)
3. Hard modes homogeneous on scale $\geq 1/k$

In heavy ion setting, (2) and (3) follow from (1)

Does *NOT* assume $\sim k$ fields are perturbative.

Hard loop approach

Treat hard modes with *kinetic theory* (Vlasov equations)

$$[D_t + \mathbf{v} \cdot D_{\mathbf{x}}]\, f(\mathbf{p}, \mathbf{x}, t) = -\frac{1}{2} \left\{ g v^{\mu} F_{\mu i}\,,\ \frac{df(\mathbf{p}, \mathbf{x}, t)}{dp_i} \right\}$$

Perturb in effect of soft modes on hard modes

$$D_t + \mathbf{v} \cdot D_{\mathbf{x}} f^a_{\mathrm{adj}} = -g v^{\mu} F^a_{\mu i} D_{p_i} f_{\mathrm{singlet}}$$

Compute induced current and feed into Yang-Mills equations

$$D_{\mu} F_a^{\nu\mu} = J_a^{\nu} = g \int_{\mathbf{p}} v^{\nu} f^a_{\mathrm{adj}}$$

Good news: $|\mathbf{v}| = 1$ so $|\mathbf{p}|$ is redundant, integrate it out.

Define $W = \int_{|\mathbf{p}|} f_{\rm adj}/g$ (roughly)

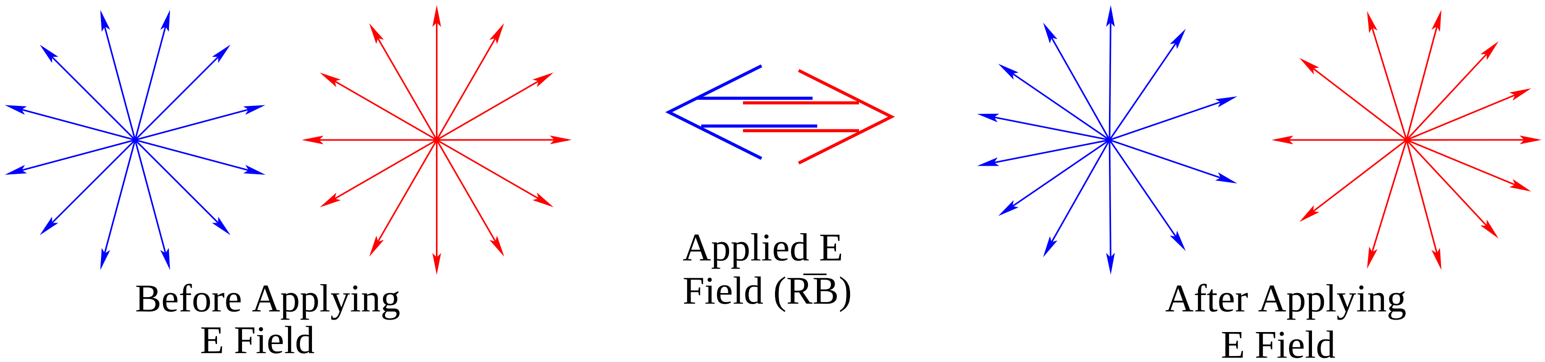
Also, write

$$g^2 C_{\rm f} \int_{\mathbf{p}} f_{\rm singlet} \delta(\mathbf{p} - \mathbf{v}|\mathbf{p}|) \equiv m_\infty^2 \Omega(\mathbf{v})$$

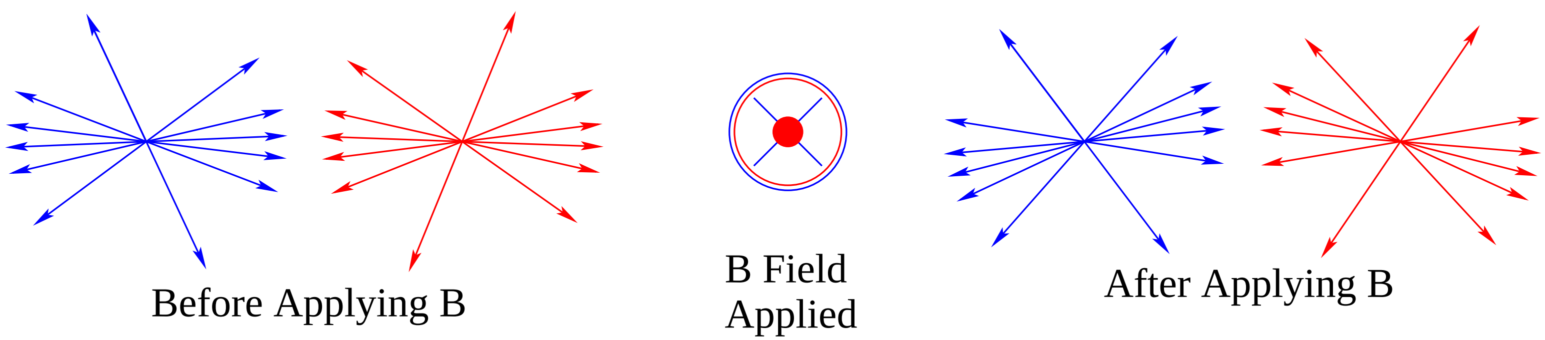
System of equations,

$$\begin{aligned}
D_t W^a(x,\vec{v}) &= -\vec{v} \cdot \vec{D} W^a(x,\vec{v}) + m_\infty^2 \, \text{Source} \\
\text{Source} &= 2\Omega(\mathbf{v}) \vec{v} \cdot \vec{E} - \vec{E} \cdot \frac{\partial}{\partial \vec{v}} \Omega(\mathbf{v}) - F_{ij} v_i \frac{\partial \Omega(\mathbf{v})}{\partial v_j} \\
D_\mu F^{\nu\mu} &= J^\nu = \int_{\mathbf{v}} v^\nu \, W(\mathbf{v})
\end{aligned}$$

Equilibrium: E fields induce color currents.



Anisotropic: B fields also induce color currents.



Always color octet. Y_{lm}: $l = 1$ (isotropic), $l > 1$ (aniso).

Lattice implementation

First: make v space discrete (otherwise, ∞ DOF/site!)

$Y_{\ell m}$ expansion, truncated at some $\ell_{\rm max}$.

$W(\mathbf{v}) \rightarrow W_{\ell m}$ and $\int_{\mathbf{v}} \ldots$ turns into ℓm sums.

Gauge fields: link variables. F^2 term: standard. SU(2).

W on sites. J on link–average of W on two ends of link.

W Eq. 1'st order: doubling. W defined only on even spacetime points.

High ℓ: "infinite" energy sink. Mock up effect by applying weak damping to high ℓ (yes we checked...).

Expect that soft field growth saturates

Because there are several unstable modes, growing at once

- Different colors
- Different k vectors

Modes interfere with each others' growth:

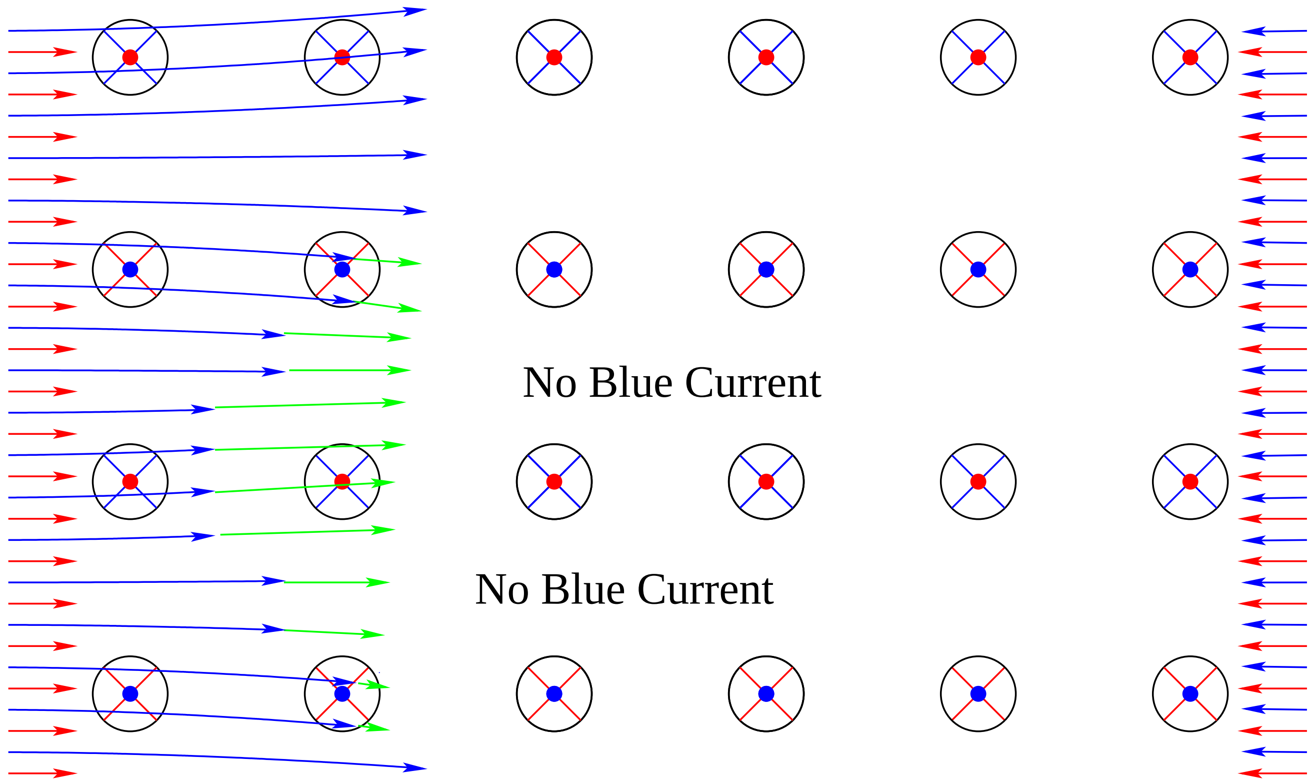
- A field color-rotates currents needed for other modes' growth
- Nonabelian interaction: soft k "scatter," moving energy into stable k wave-vectors.

Requires many unstable modes: this requires

- Large volume (more possible k)
- Not too-long exponential growth only few fast-growing k become large

Color rotation

Suppose another field, not shown, causes colors to rotate:



Current does not support B. Growth stops/is inhibited

Arnold and Lenaghan proposal

Hard gauge fields induce plasma masses for each other.

Suggests: large soft field makes other soft fields massive.

Large field doesn't induce mass for itself: behavior of *one* color is abelian

Idea: growth of 1 color "gets ahead." Acts as mass, squashing other colors.

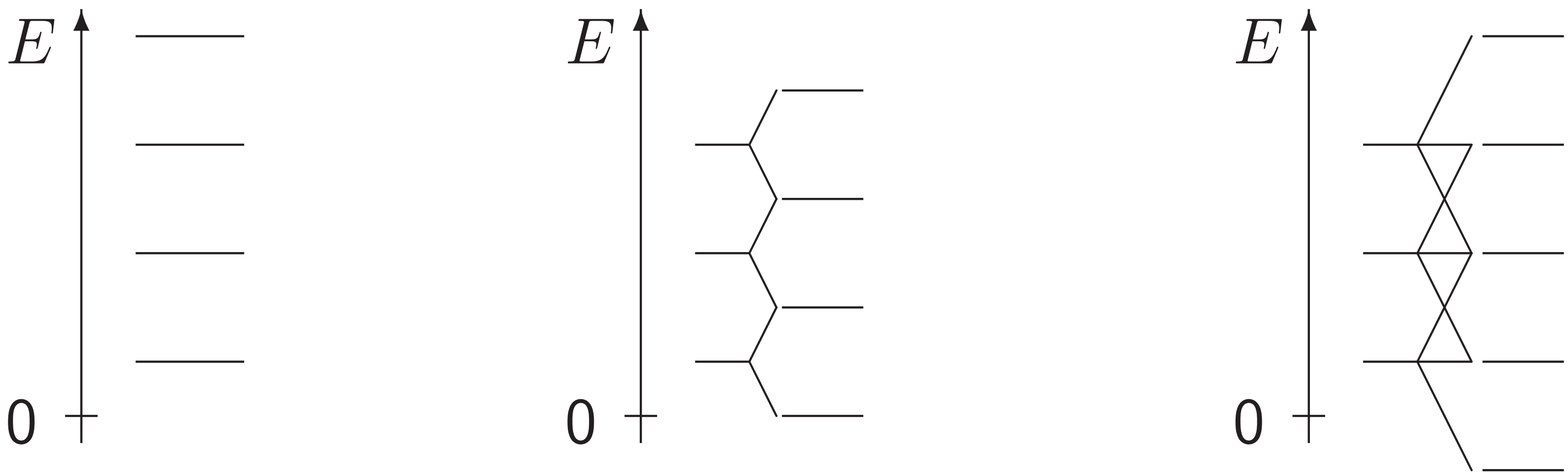
System abelianizes, growth continues.

Behavior observed in 1D3V simulations.

Except, One B field *causes* others to grow

B field splits states into Landau levels. Split by $\vec{s} \cdot \vec{B}$.

Spin 0 | Spin $\frac{1}{2}$ | Spin 1

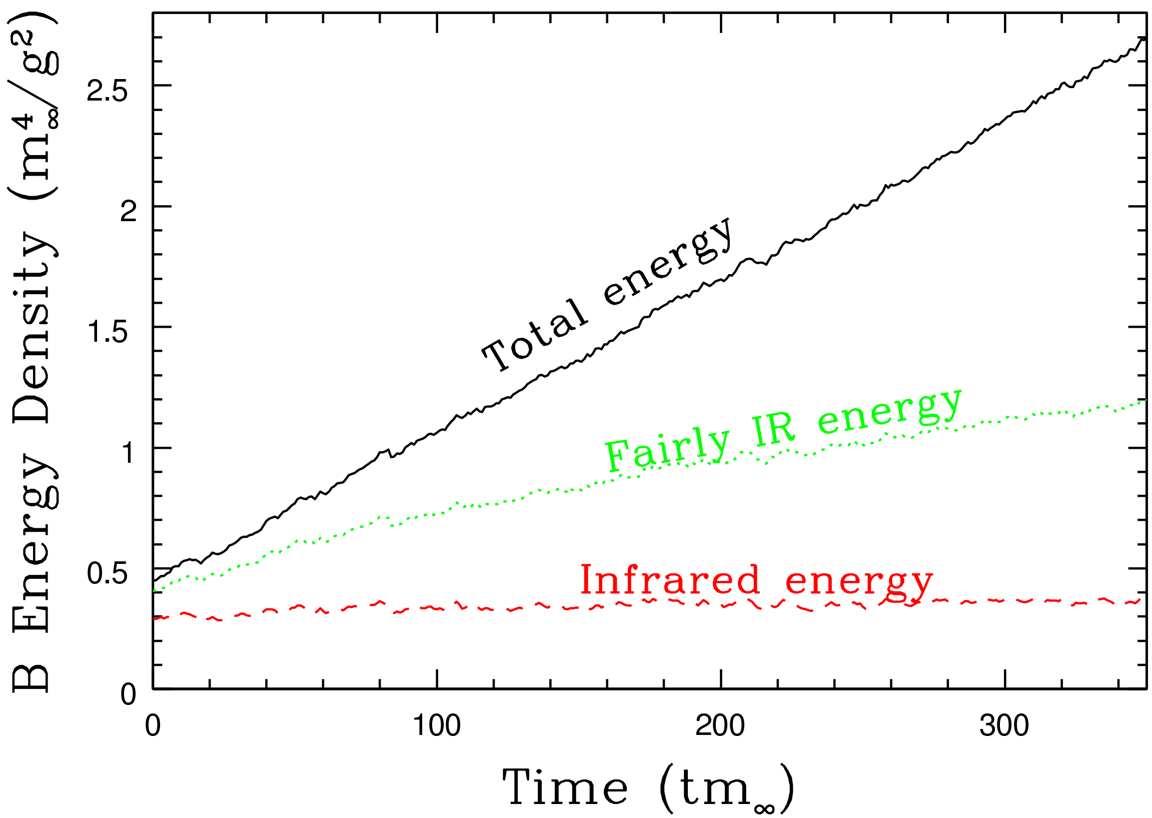


All stable | One $E = 0$ | One is unstable

Nielsen-Olesen instability.

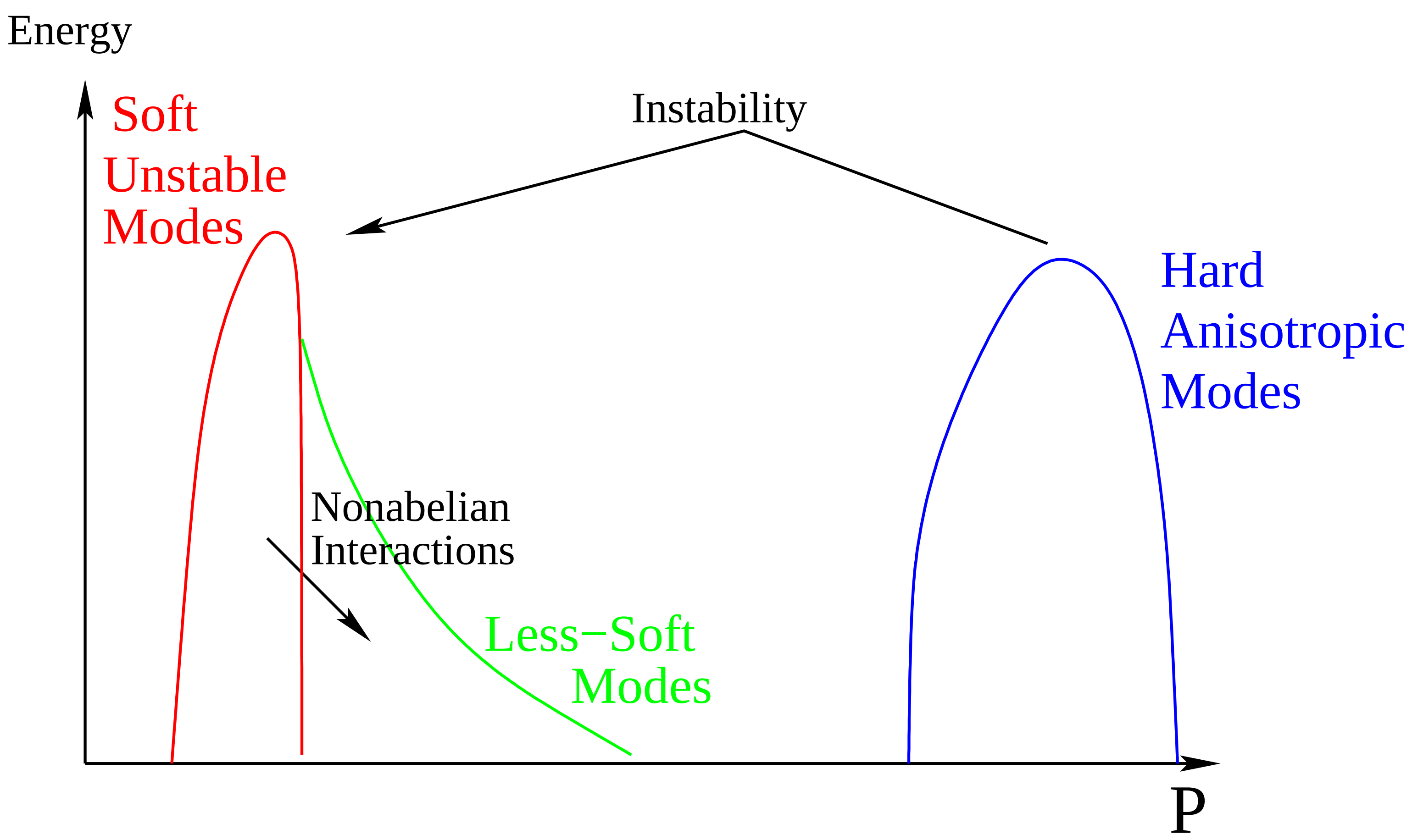
Energy in gauge fields grows linearly with time

Field smearing lets us see how much is very IR energy.

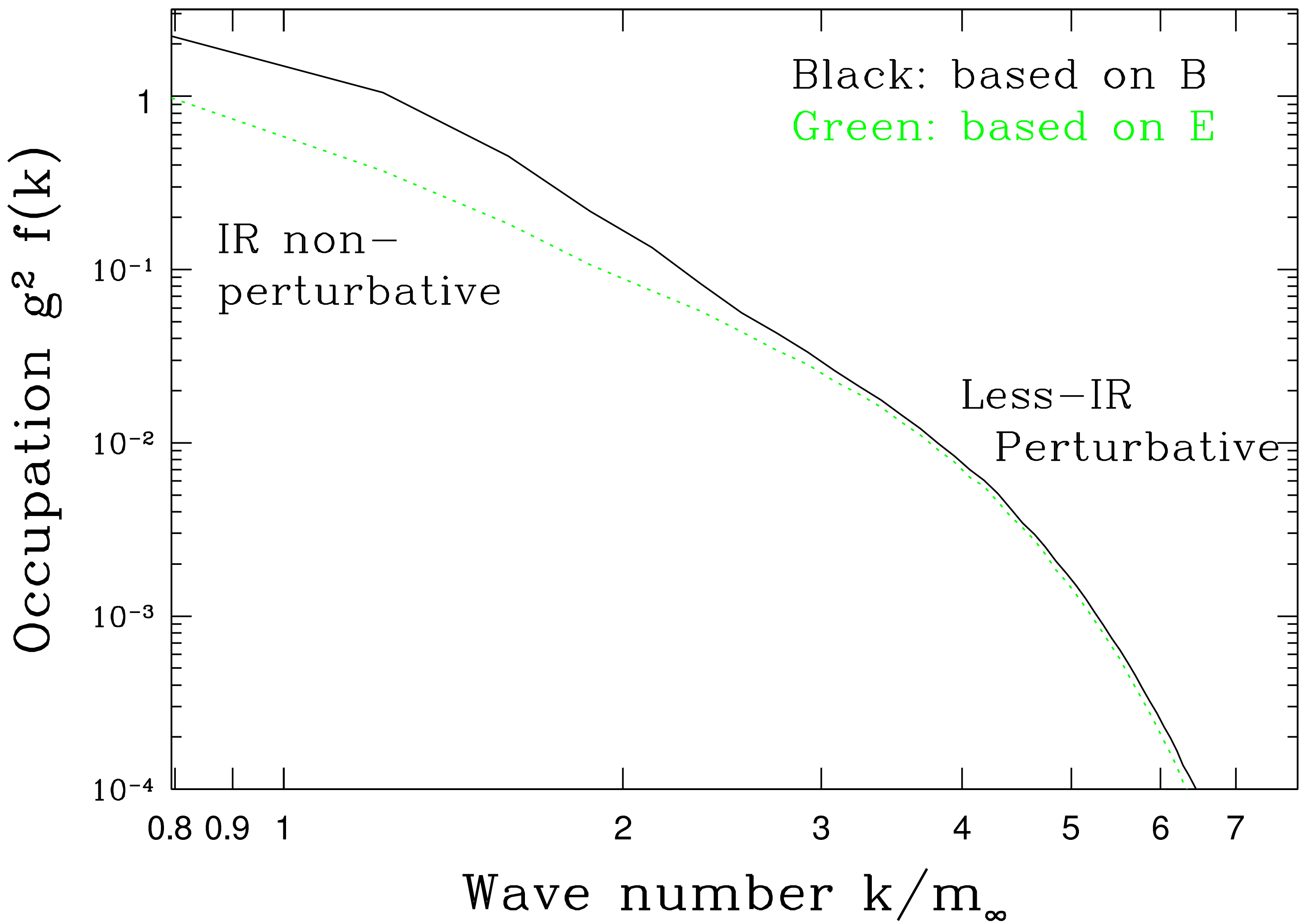


Very soft fields constant. Medium-soft grow slower.

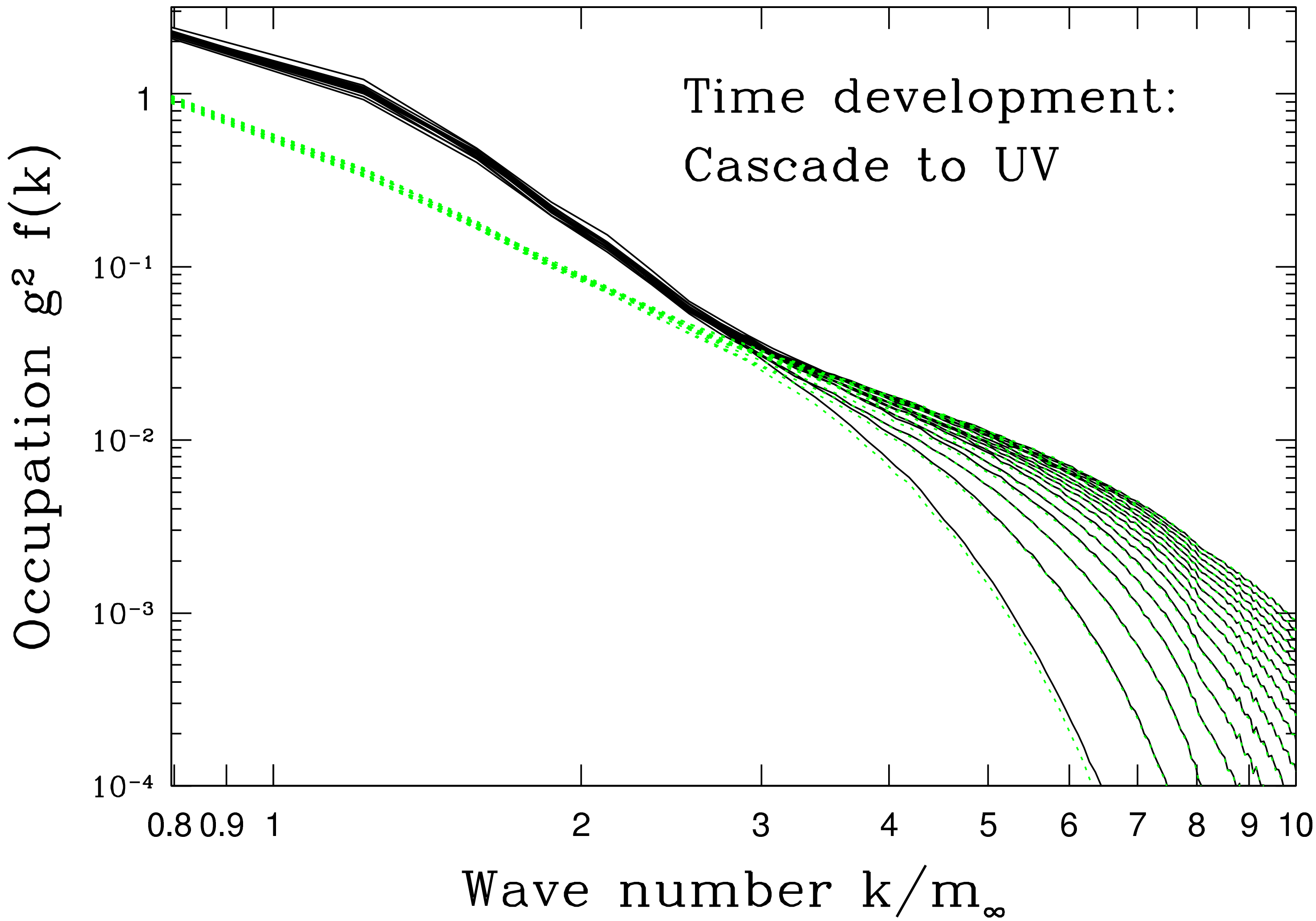
Instability pumps soft modes. Nonabelian interaction cascades energy into less-soft modes.



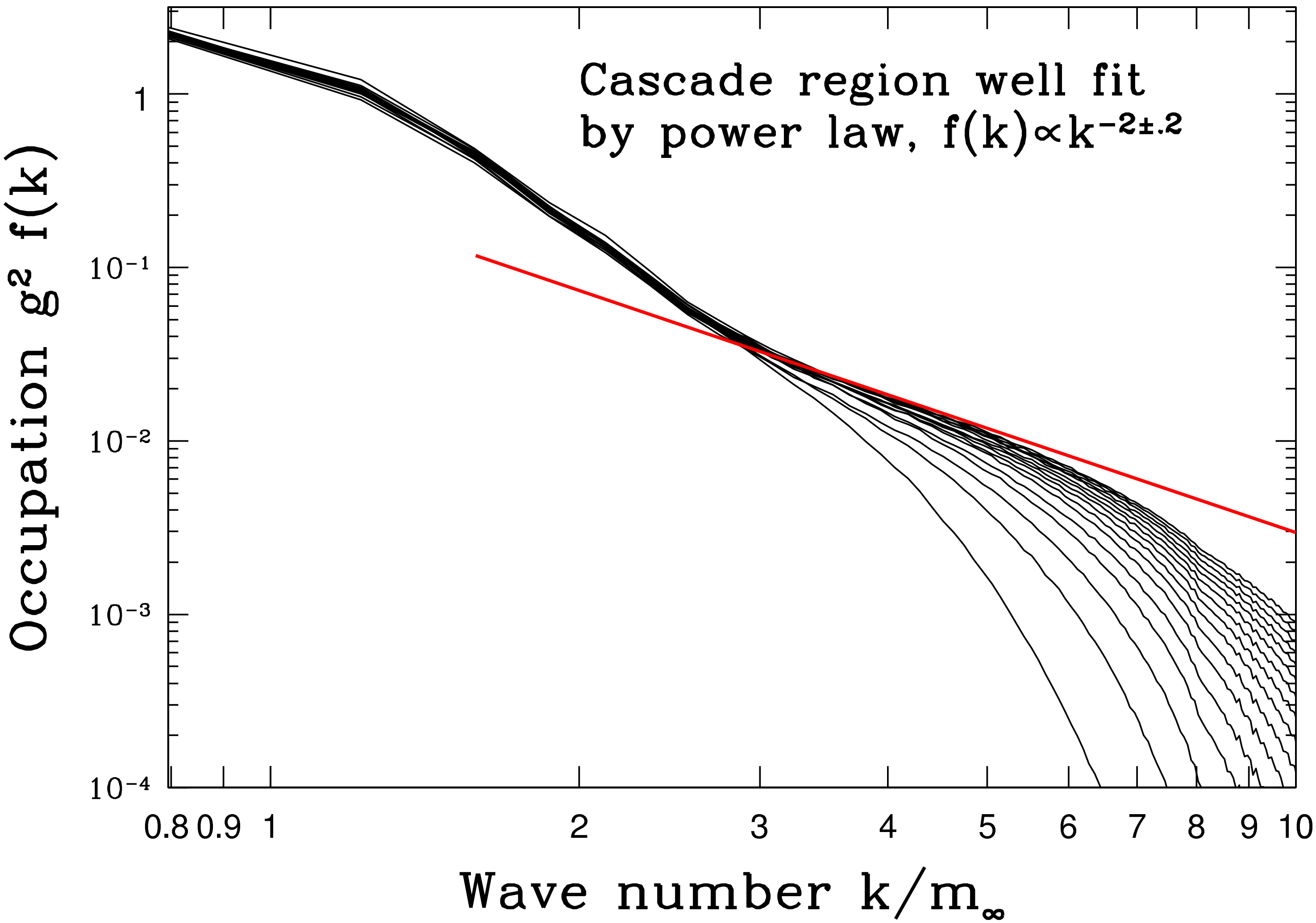
Coulomb gauge power spectrum: Initially



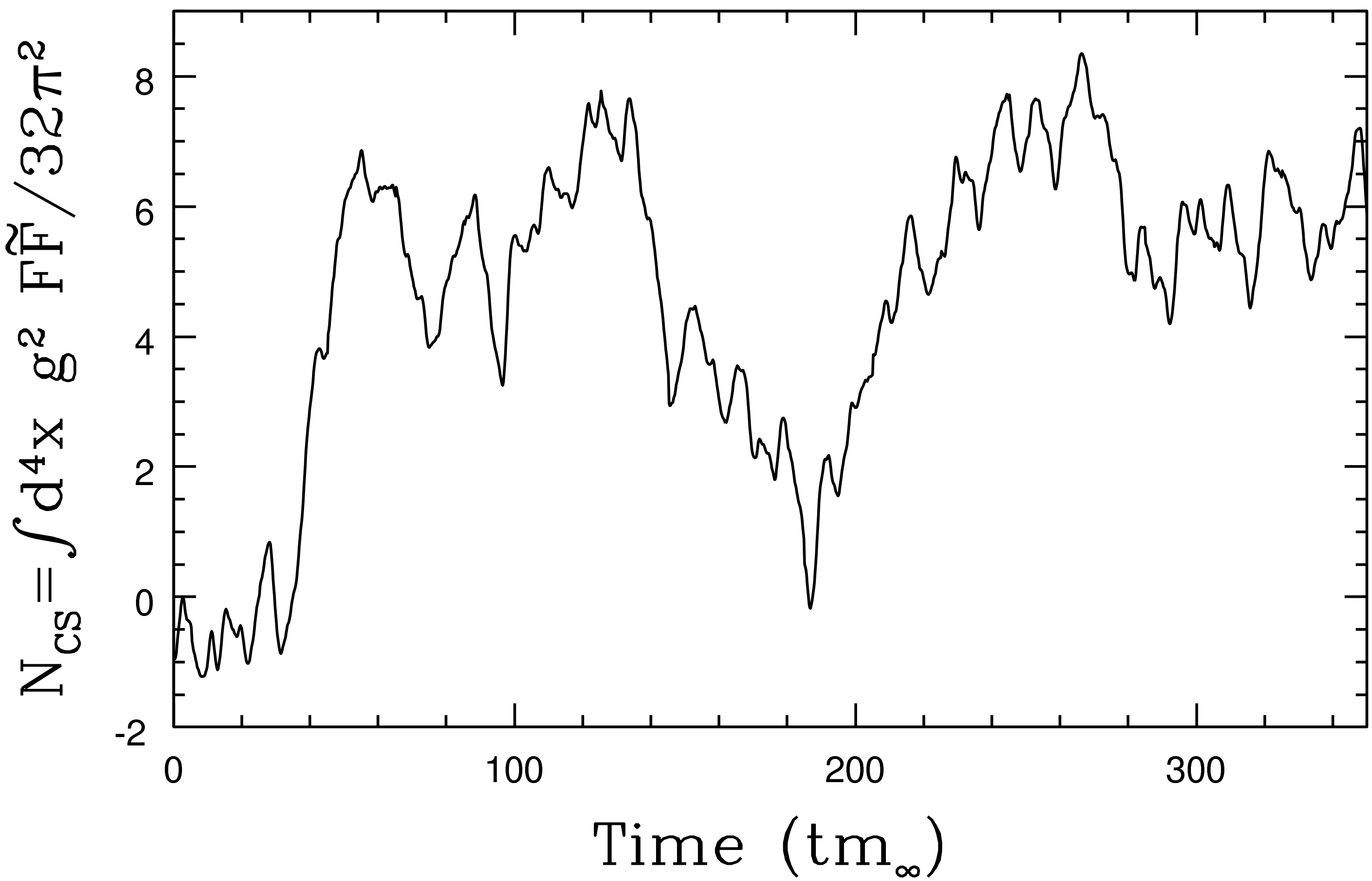
Time development of Coulomb gauge spectrum



Power-law behavior with moving cutoff

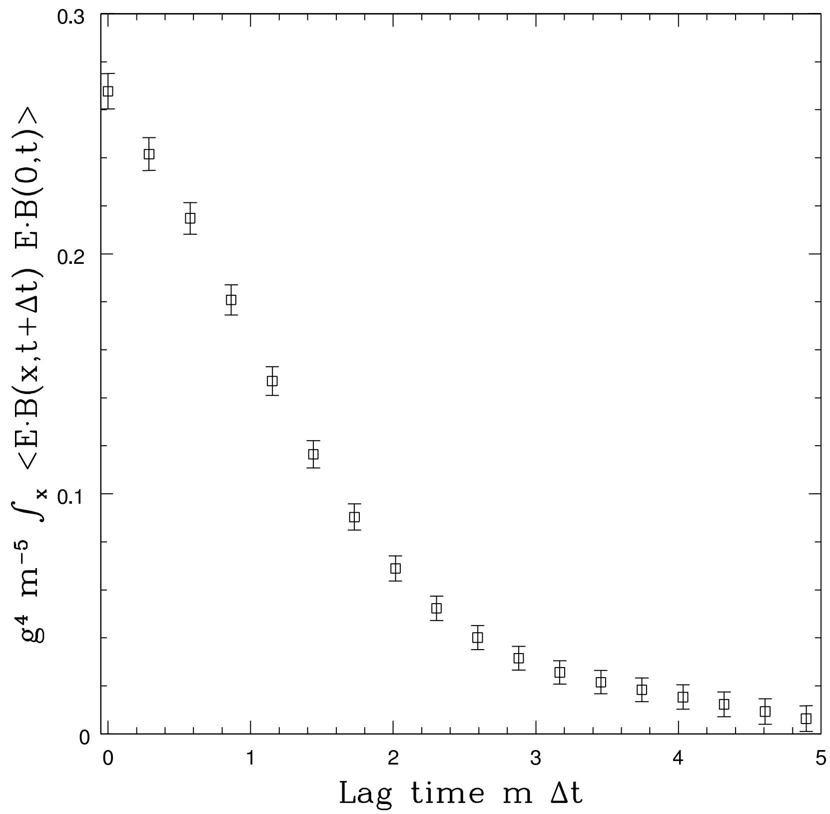


Soft nonabelian fields are large and randomly fluctuating, as seen in chaotic evolution of Chern-Simons number:



IR field correlation time

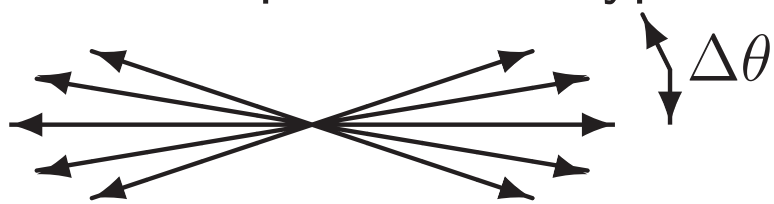
Proxy for Time for IR fields to re-arrange–$E \cdot B$ coherence



Comparable to the intrinsic $1/m$ scale (reasonable)

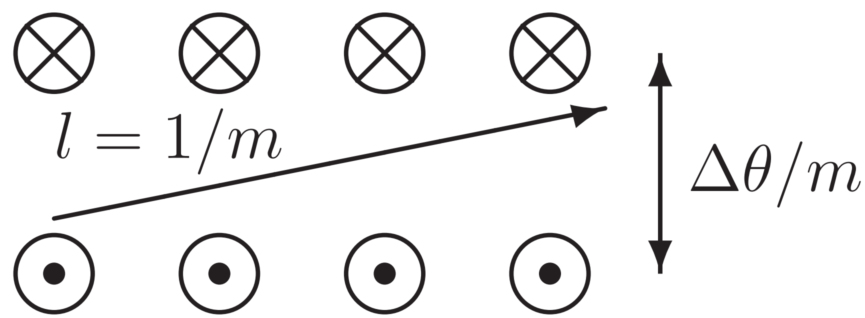
What about strong anisotropy?

Suppose hard modes are planar with typical deviation $\Delta\theta$.

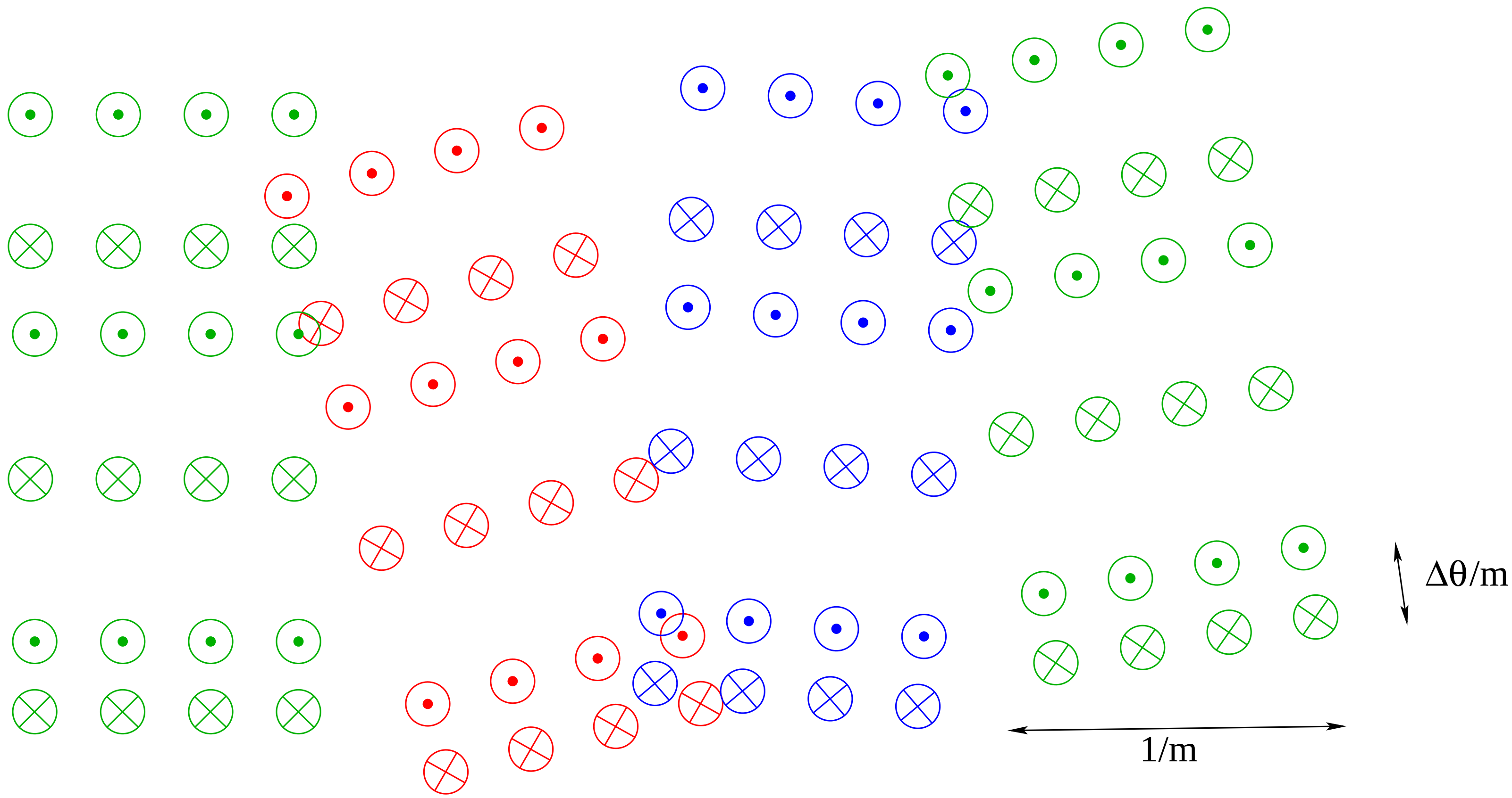


B field is unstable if all particles are in same sign field for time $t \sim 1/m$. This requires

$$k_\perp < m \quad \text{and} \quad k_z < \frac{m}{\Delta\theta}$$



Typical unstable config. for large anisotropy



B fields change fast vertically, slowly horizontally.

How big does field grow?

What cuts off field growth?

Hard mode color rotation less than every $1/m$ length:

$$gA \sim m \to B = \nabla A \sim k\frac{m}{g} \sim \frac{m^2}{g\Delta\theta}$$

Nonabelian mutual interactions: nontrivial Wilson line over transverse length also $\sim 1/m$ gives $A \sim m/g$.

$$A \sim \frac{m}{g} \qquad \epsilon = B^2 \sim \frac{m^4}{g^2\Delta\theta^2} \qquad E \sim D_t A \sim \frac{m^2}{g}$$

since the *time* scale for change is still $1/m$!

B larger, but E same size, as $O(1)$ anisotropy.

Momentum randomization of hard modes:

$$\frac{dp_{\rm perp}^2}{dt} \sim (gB)^2 \delta t \sim g^2 B^2 \frac{1}{m} \sim \frac{m^3}{\Delta\theta^2}$$

Time coherence $\delta t = 1/m$ only because moving in-plane

Cascade particle energy gain: E field.

$$\frac{dp_{\parallel}^2}{dt} \sim (gE)^2 \delta t \sim g^2 E^2 \frac{\Delta\theta}{m} \sim m^3 \Delta\theta$$

Random angle: E field adds coherently for shorter times!

Cascade by making new quanta: mismatch of k and ω

cascade=plasmons, $\omega > k$. Soft modes $k \sim m/\Delta\theta$, $\omega \sim m$.

Large anisotropy: bigger B, same sized cascade.

Conclusions

- Anisotropy $\longrightarrow$ plasma instability
- Instability cut off by nonabelian effects
- Lattice treatment via Hard Loop approx. is possible
- Energy feeds into a gauge field cascade
- Strong anisotropy $\longrightarrow$ bigger fields, same cascade

Most pressing problems:

- understanding large anisotropy cascade
- quantitative question of (un?)importance of instability

Things still to do

Do instabilities have any relevance?

- Compare "kicks" on hard modes to hard-hard scattering
- Understand bremsstrahlung emission due to kicks on hard modes
- Understand startup time–are instabilities just too late?

See work by

- Romatschke, Venugopalan
- Strickland, Dumitru, Nara
- Bödeker, Rummukainen